

# Trends in Non-Linear Analysis 2015

Organizers:

José Matias – CAMGSD, Instituto Superior Técnico, Lisboa, Portugal

Marco Morandotti – SISSA, Trieste, Italy

SISSA, Trieste, Italy – 1-3 July, 2015

## Timetable

	Wednesday - July 1	Thursday - July 2	Friday - July 3
9:00-9:30	<i>registration</i>		
9:30-10:30	<b>Alouges</b>	<b>Stefanelli</b>	<b>Angelillo</b>
10:30-11:00	<b>Lucardesi</b>	<b>Rinaldi</b>	<b>Giraldi</b>
11:00-11:30	<i>coffee break</i>	<i>coffee break</i>	<i>coffee break</i>
11:30-12:30	<b>Reali</b>	<b>Pandolfi</b>	<b>Mora</b>
12:30-13:00	<b>Solombrino</b>	<b>Carita</b>	<b>De Luca</b>
13:00-15:00	<i>lunch break</i>	<i>lunch break</i>	<i>lunch break</i>
15:00-16:00	<b>Ferreira</b>	<b>Pedregal</b>	<b>Farkas</b>
16:00-16:30	<b>Hayrapetyan</b>	<b>Barchiesi</b>	<b>Shum</b>
16:30-17:00	<i>coffee break</i>	<i>coffee break</i>	<i>coffee break</i>
17:00-17:30	<b>Monsaingeon</b>	<b>Zwicknagl</b>	<b>Davoli</b>
17:30-18:00	<b>Evers</b>	<b>Piovano</b>	<b>Gadélha</b>
20:30		<i>social dinner</i>	<i>SISSA party</i>

The social dinner will take place at *Peperino Pizza & Grill*, Via del Coroneo, 19, Trieste.

## Abstracts

**François Alouges** :: École Polytechnique, Palaiseau

***The sparse cardinal sine decomposition and applications***

When solving wave scattering problems with the Boundary Element Method (BEM), one usually faces the problem of storing a dense matrix of huge size which size is proportional to the (square of) the number  $N$  of unknowns on the boundary of the scattering object. Several methods, among which the Fast Multipole Method (FMM) or the  $H$ -matrices are celebrated, were developed to circumvent this obstruction. In both cases an approximation of the matrix is obtained with a  $O(N \log(N))$  storage and the matrix-vector product has the same complexity. This permits to solve the problem, replacing the direct solver with an iterative method.

The aim of the talk is to present an alternative method which is based on an accurate version of the Fourier based convolution. Based on the non-uniform FFT, the method, called the sparse cardinal sine decomposition (SCSD) ends up to have the same complexity than the FMM for much less complexity in the implementation. We show in practice how the method works, and give applications in as different domains as Laplace, Helmholtz, Maxwell or Stokes equations.

This is a joint work with Matthieu Aussal.

**Maurizio Angelillo** :: Università di Salerno

***Singular Stress Fields for Masonry-like Vaults: Where Mathematics Meets History***

It is maybe a trivial remark, saying that the vast majority of masonry structures (excluding tall towers) exhibit an extraordinary stability under the effect of age and settlements and even under the repeated action of strong winds and heavy earthquakes. The reason of this unperturbed stability stems from the so-called strength by shape that is typical also of structures carrying tangent internal forces, such as cords, nets and membranes, that is unilateral structures. If unperturbed-unilateral stability for masonry, does not mean plainly dumb or boring stability, this is actually due to the curved structural elements (arches, vaults and domes) that started to appear systematically in masonry Architecture since the ancient Rome. In this presentation the unilateral model for masonry is applied to explain the equilibrium of some beautiful and puzzling structures, that is a Gothic cross vault with ribs, and two spiral stairs one of the end of XVII century and the other one of the beginning of the XIX century.

The unilateral model, which appears as the clue of structural interpretation behind the design of the great Architecture masterpieces of the past, was first rationally introduced in the scientific community by Heyman in 1966, with his milestone paper *The stone skeleton*. The model is based on three crude assumptions:

1. The material does not bear the slightest tensile load (unilateral assumption) and then can detach, at zero energy, along any internal interface (fracture line).
2. The material is indefinitely resistant to compression: that is the mean compressive stresses are so low that crushing of the material is not an issue.
3. There is no possibility of sliding along a fracture line (infinite friction).

The unilateral assumption is an extreme approximation for the brittleness of the material under tensile loads, brittleness being responsible for the softening behaviour of masonry at the macroscopic level.

No-sliding is equivalent to assume infinite friction, and friction and sliding are the basic mechanisms in block-block, and block-mortar-block interactions. Understanding toughness and friction is then obviously a necessary step toward the goal of obtaining a detailed masonry description. Anyone working at some depth in material engineering knows that fracture and friction are still the most difficult challenges of modern Mechanics.

The main strength of the simplified unilateral model of Heyman, which assumes zero toughness and infinite friction, is indeed its ability, while excluding these two tough guys, of being still able to make sound predictions on masonry behaviour.

With Heyman's assumptions, the equilibrium of a structure composed of this ideal masonry material, can be studied with Limit Analysis. Displacement discontinuities and concentrated stresses are inherent to the unilateral behaviour, therefore, analytical and numerical tools must be introduced that can handle the unilateral No-Tension model with singular stress and strain fields, within the framework defined by the two theorems of Limit Analysis.

In particular, the present study is concerned with the application of the safe theorem of limit analysis to masonry-like vaults, that is, curved constructions modelled as continuous unilateral bodies.

On allowing for singular stresses in the form of line or surface Dirac deltas, statically admissible stress field concentrated on surfaces (and on their folds) lying inside the masonry, are considered. Such surface and line structures are unilateral membranes/arches, whose geometry is described *à la* Monge, and their equilibrium can be formulated in Pucher form. It is assumed that the load applied to the vault is carried by such a (possibly folded) membrane structure  $S$ . The geometry of the membrane  $S$ , that is of the support of the singularities, is not fixed, in the sense that it can be displaced and distorted, provided that one keeps it inside the masonry.

In the case of pure vertical loading, the problem is reduced to a singular partial differential equation of the second order where the shape  $f$  and the stress function  $F$  appear symmetrically. The unilateral restrictions require that the membrane surface lies in between the extrados and intrados surfaces of the vault and that the stress function be concave.

Such a constraint is, in general, not satisfied on a given shape for given loads: in such a case, the shape has to be modified to fit the constraint. In a sense, the unilateral assumption renders the membrane an underdetermined structure that must adapt its shape in order to satisfy the unilateral restrictions.

Three particular case studies are analyzed to illustrate the method: the cross vaults of the Gothic Cathedral of Caserta, the Baroque spiral stair of San Domingos de Bonaval and the Modern timber spiral stair built by the Guastavinos in New York.

**Marco Barchiesi** :: Università di Napoli Federico II

***Local invertibility in Sobolev spaces and applications***

I will discuss the local invertibility of Sobolev maps that are regular, in the sense that they undergo no cavitation. I will show that the invertibility is stable under the weak convergence, and I will use this property to provide the well-posedness of a nonlinear model for nematic elastomers.

**Graça Carita** :: Universidade de Évora

***Relaxation in  $BV \times L^p$***

An integral representation result is obtained for the relaxation of a class of energy functionals depending on two vector fields  $u$  in  $BV$  and  $v$  in  $L^p$  with  $1 < p \leq +\infty$ . This energy have been introduced to deal with equilibria for systems depending on chemical composition and elastic strain.

In this work we add the dependence on the spatial variable  $x$  and the deformation  $u$  to the functional. Also the quasiconvexity assumption is dropped.

This result may also find applications in the framework of image decomposition models.

This is a joint work with Elvira Zappale.

**Elisa Davoli** :: Universität Wien

***Multiscale homogenization in Kirchhoff's nonlinear plate theory***

The interplay between multiscale homogenization and dimension reduction for nonlinear elastic thin plates is analyzed in the case in which the scaling of the energy corresponds to Kirchhoff's nonlinear bending theory for plates. Different limit models are deduced depending on the relative ratio between the thickness parameter  $h$  and the two homogenization scales  $\varepsilon$  and  $\varepsilon^2$ .

This is a joint work with Laura Bufford and Irene Fonseca.

**Lucia De Luca** :: Technische Universität München

***Dynamics of Discrete Screw Dislocations in SC, BCC, FCC and HCP crystals: A variational approach***

Dislocations are 1D defects in the periodic structure of crystals, whose presence is believed to be the main mechanism of plastic deformation in metals. Using a variational approach based on  $\Gamma$ -convergence and on the minimizing movements scheme, we study the dynamics of a special class of dislocations, the screw ones, in Simple Cubic (SC), Body Centered Cubic (BCC), Face Centered Cubic (FCC) and Hexagonal Close Packed (HCP) crystals.

The results we will present are fruit of an ongoing work with R. Alicandro, A. Garroni and M. Ponsiglione.

**Joep Evers** :: Technische Universiteit Eindhoven

***Flux boundary conditions for measure-valued evolutions***

In this talk, I want to impose a flux boundary condition on a measure-valued mass evolution problem posed on a bounded interval. We start from a semigroup on measures that accounts for (1) the dynamics inside the interval, and (2) the stopping of trajectories on the impermeable boundaries of the interval. Additionally, we introduce a thin layer near the boundary in which absorption of mass takes place, and let the width of this layer tend to zero. We derive the convergence rate for this approximation procedure as well as the structure of the flux boundary condition in the limit problem.

The above procedure works for the simplistic case in which the velocity field is prescribed. The second part of my talk is devoted to solution-dependent velocity fields, which are relevant in view of mutual interactions.

Joint work with: Adrian Muntean (Eindhoven, NL) and Sander Hille (Leiden, NL)

**Jozsef Farkas** :: University of Stirling

***Steady states of some nonlinear PDEs modelling structured populations***

In this talk we will discuss the question of existence of non-trivial steady states of some partial differential equations, which are intended to model structured populations. We reformulate the steady state problem as an abstract eigenvalue problem coupled with a fixed-point problem. This approach allows us to formulate biologically relevant conditions for the existence of a positive steady state. We will discuss the steady state problem for models with monotone infinite dimensional nonlinearities, as well as for models with non-monotone finite dimensional nonlinearities.

This talk is based on joint work with Angel Calsina (Universitat Autònoma de Barcelona).

**José Augusto Ferreira** :: Universidade de Coimbra

***Mathematical modeling and numerical simulation of non-Fickian diffusion phenomena***

When a fluid penetrates a viscoelastic polymer its transport is not accurately described by a classical diffusion-reaction equation. A gap between experimental and simulation results obtained using Fickian diffusion models has been observed. Brownian motion of fluid molecules should be connected by a term representing the stress response of the material to the deformation of the incoming fluid. The fact that the classical diffusion equation does not accurately describe transport phenomena is felt not only in polymer sciences but also in other scientific domains, like diffusion processes in porous media. In recent years, some attempts have been made to model mathematically such behaviors using integro-differential equation of Volterra type. The development of efficient and accurate numerical methods to simulate the new integro-differential models has attracted the attention of several researchers and a significant number of contributions can be found in the literature. This talk is focused mainly on the mathematical modeling and numerical simulation of non-Fickian diffusion phenomena in different scenarios.

**Hermes Gadêlha** :: University of York

***The specious material response of cross-linked filament bundles and flagella***

Recent observations of flagellar counterbend in sperm show that the mechanical induction of curvature in one part of a passive flagellum induces a compensatory countercurvature elsewhere. This apparent paradoxical effect cannot be explained using the standard elastic rod theory of Euler and Bernoulli, or even the more general Cosserat theory of rods. Here, we develop a mechanical model capable of predicting the curvature reversal events observed in eukaryotic flagella. This is achieved by allowing the interaction of deformations in different material directions, by not only accounting for structural bending, but also the elastic forces originating from the cross-linking mechanics. Large amplitude configurations can be described analytically and an excellent match between the model and the observed counterbend deformation was found. This allowed a simultaneous estimation of multiple sperm flagellum material parameters, namely the cross-linking sliding resistance, the bending stiffness and the sperm head junction compliance ratio. Our analysis demonstrates that the counterbend emerges as a fundamental property of sliding resistance, which also suggests that cross-linking proteins may contribute to the regulation of the flagellar waveform in swimming sperm via counterbend mechanics. Finally, we investigate how the counterbend-type dynamics in sperm flagella is affected by viscous dissipation.

**Laetitia Giraldi** :: ENSTA – Paristech

***Biomechanical pattern formation for cell growth***

In this talk, we introduce a cell growth model that combines the bio-mechanic growth effect of a viscoelastic membrane and the geometry of the cell wall. More specifically, we consider that the density of available building materials depends on the curvature of the cell wall. This particular assumption leads to introduce anisotropic growth in the model. We show that the dynamics of the cell wall is described as a system of coupled PDEs including a convection-diffusion equation and a generalized Laplace equation with Neumann boundary conditions. Using a linear stability analysis, we identify which structure parameters make the membrane wall unstable around a spherical shape. In particular, we show that the coupling between the two equations has to be highly non-linear to make the system unstable around the spherical shape. Finally, we present numerical simulations which confirm this theoretical stability analysis.

**Gurgen (Greg) Hayrapetyan** :: Ohio University

***Stability and evolution of interfaces in amphiphilic systems***

Amphiphilic systems arise as cellular membranes and ion channels in biological systems and drive the self-assembled nanoscale network morphologies, which are essential to the efficiency of energy conversion devices such as fuel cells, Lithium ion batteries, and dye sensitized solar cells.

We discuss problems in interface evolution within amphiphilic systems, including the rigorous reductions of gradient flows of higher-order phase field energy functionals that model phase separation.

**Ilaria Lucardesi** :: SISSA

***The wave equation on domains with cracks growing on a prescribed smooth path***

Given  $\Omega \subset \mathbb{R}^N$  an open bounded domain with Lipschitz boundary, and  $\{\Gamma_t\}_{t \in [0, T]}$  an increasing family of subsets of  $\Omega$ , we analyze the scalar wave equation  $u_{tt} - \Delta u = f(t)$  in the time varying domain  $\Omega \setminus \Gamma_t$ , prescribing a zero Neumann condition on  $\Gamma_t$ .

Here we assume that the crack position, described by  $\Gamma_t$ , is known a priori, and that the fracture grows on a prescribed  $(n - 1)$ -manifold of class  $C^2$ .

Our approach relies on a change of variables: recasting the problem on the reference configuration  $\Omega \setminus \Gamma_0$ , we are led to consider a hyperbolic problem of the form  $v_{tt} - \operatorname{div}(A(t)\nabla v) + b(t) \cdot \nabla v + c(t) \cdot \nabla v_t = g(t)$  in  $\Omega \setminus \Gamma_0$ , again with zero Neumann condition on  $\Gamma_0$ . Here the coefficients of the PDE depend on time, but the problem is now set on the cylindrical domain  $[0, T] \times (\Omega \setminus \Gamma_0)$ .

Under suitable assumptions on the velocity and the acceleration of the crack propagation, we prove existence and uniqueness of weak solutions for both formulations. Moreover, we provide an energy equality which gives, as a byproduct, the continuous dependence of the solution with respect to the data. This is a joint work with G. Dal Maso.

**Leonard Monsaingeon** :: Instituto Superior Técnico

***A new optimal transport distance between nonnegative measures***

In this talk, I will introduce a new distance between nonnegative finite Borel measures in  $\mathbb{R}^d$ . The distance is constructed by a Lagrangian approach (minimization of an action functional), which is similar to the celebrated Benamou-Brenier formula (dynamical representation of the quadratic Kantorovich-Rubinstein-Wasserstein distance between probability measures). Compared to the classical theory of optimal transportation of probability measures, our new distance has the advantage of allowing for mass changes, and the theory does not require any finite moments or decay at infinity.

Among other results, we obtain for the resulting metric space: completeness, local equivalence with other distances, lower semi-continuity properties, existence of geodesics, and characterization of Lipschitz curves.

If time permits, I will discuss the application to a fitness-driven model of population dynamics: once suitably interpreted as a gradient flow with respect to our metric, we show that the model satisfies exponential convergence to the unique steady state with explicit rates. This work in progress is joint with D. Vorotnikov (Univ. Coimbra) and S. Kondratyev (Univ. Coimbra).

**Maria Giovanna Mora** :: Università di Pavia

***Homogenization of time-dependent systems of dislocations***

It is well known that plastic, or permanent, deformation in metals is caused by the concerted movement of many curve-like defects in the crystal lattice, called dislocations. What is not yet known is how to use this insight to predict behaviour at continuum scales. In this talk I will present a rigorous upscaling result for a discrete system of moving edge dislocations in two dimensions with slip-plane confinement. In the continuum limit we obtain an evolution law for the dislocation density. This is a joint work with Lucia Scardia (Bath) and Mark A. Peletier (Eindhoven).

**Anna Pandolfi** :: Politecnico di Milano

***Patient specific models of the eye refractive system***

The growing demand for permanent refractive corrections and laser surgical procedures, such as PRK and LASIK, has increased the importance of corneal topography data in both clinical and research settings. Within the past 10 years, corneal topographers have been converted from elaborate and costly devices used exclusively for clinical research to in-office tools that optometrists use daily. Along with advances in computerization and software development, topographers have become smaller, more compact, multifunctional, easy-to-handle, less expensive and more precise. Modern topographers use advanced techniques which do not require anaesthetics nor direct contact with the cornea. The software associated to advanced topographers provides the coordinates of points lying on the anterior surface of the cornea and the corneal thickness at the same points. By means of mathematical interpolation of these points, it is possible to reconstruct the accurate geometry of the corneal anterior and posterior surfaces in preoperative and postoperative conditions.

The current ophthalmologic literature testifies a growing interest in numerical models of the anterior segment of the eye. The very first contributions worked on idealized geometries and isotropic materials and addressed the simulation of refractive procedures. Successive works included the underlying anisotropic microstructure of the stroma and tried to focus on the qualitative evaluation of refractive surgery outcomes, or to explain the reliability of intraocular pressure measurements. Examples of numerical applications that evaluate the qualitative mechanical response of the anterior chamber of the eye to different actions can be found in the recent literature. If the eye's geometry is known and sufficient information on the material properties is available, it is possible to create a personalized numerical model of the eye that possesses predictive (therefore quantitative) abilities.

In view of obtaining a predictive numerical model of the anterior segment of the eye, it is necessary to use patient-specific geometry and patient-specific material models. Modern diagnostic

instruments for ophthalmology acquire the tridimensional geometry of the whole anterior chamber of eye; therefore, customized purely geometrical models are already available and standard stress analysis codes have the potential to predict the qualitative response to refractive surgery. Regrettably, patient-specific material models are not available yet, given the difficulty to identify the material properties of the different parts of the anterior segment of the eye by means of non destructive in-vivo tests. Research is moving fast in view of tackling this missing point, trying to combine innovative experimental methods with inverse analysis in advanced identification procedures. A promising testing procedure is the air-puff tonometry, combined with imaging and numerical analysis, but the technique is not fully developed yet and needs to be coupled to different material models in order to reproduce the behaviour of individual corneas.

In this talk, I will describe the satisfying effort that has been carried out in order to develop an advanced numerical procedure that, starting from images of the anterior chamber of the eye, builds an accurate, patient-specific geometrical model of the human cornea. The model is used in finite element static stress analyses that estimate, by means of a simplified identification procedure, the material properties of the eye according to the chosen material model. The finite element code uses improved models of statistically distributed fiber materials, applied over a particular organization of the collagen fibrils in the cornea. The material model has been proved to be accurate, robust, and efficient.

The code is the fruitful result of the collaborations with Irene Simonini, Paolo Sanchez, Marcello Vasta, Alessio Gizzi, Maurizio Angelillo, Gerhard Holzapfel, Federico Manganiello and Giorgio Fotia.

**Pablo Pedregal** :: Universidad de Castilla-La Mancha

***Peridynamics through  $\Gamma$ -convergence***

Peridynamics is a nonlocal model in Continuum Mechanics, and in Elasticity, introduced by Silling (2000).

The nonlocality is reflected in the fact that points at a finite distance exert a force upon each other.

If, however, those points are more distant than a characteristic length called horizon, it is customary to assume that they do not interact. We work in the variational approach of time-independent deformations, according to which, their energy is expressed as a double integral that does not involve gradients.

We prove that the  $\Gamma$ -limit of this model, as the horizon tends to zero, is the classical model of hyperelasticity. We pay special attention to how the passage from the density of the non-local model to its local, nonlinear elastic counterpart, takes place. Depending on time, a short discussion about weak lower semicontinuity for the non-local model will also be addressed. Joint work with J. C. Bellido and C. Mora-Corral.

**Paolo Piovano** :: Universität Wien

***Wulff shape and isoperimetric characterization of crystals***

In this talk the problem of analytically explaining why particles at low temperature arrange in periodic lattices will be considered and the emergence of the Wulff shape of crystals will be investigated. Ground states of phenomenological energies accounting for two-body and three-body short-ranged interactions will be shown to be connected subsets of the reference lattice, and their energies will be exactly quantified.

As the energy favors particle bonding and ‘boundary’ particles have in general less bonds, ground states are intuitively expected to have minimal ‘perimeter’, or maximal ‘area’. This intuition will be verified by introducing a suitable notion of perimeter and area of configurations, and by showing that ground states are characterized as those configurations which realize equality in a discrete isoperimetric inequality.

In view of this characterization the emergence of a macroscopic Wulff shape as the number of particles grows will be established, and ground states will be shown to deviate from the asymptotic Wulff shape at most by  $O(n^{3/4})$  particles. This result nicely reflects the inherent multiscale nature of the crystallization phenomenon.

**Alessandro Reali** :: Università di Pavia

***Isogeometric Analysis: An innovative paradigm for Computational Mechanics***

Isogeometric Analysis (IGA) is a recent simulation framework, originally proposed by TJR Hughes and coworkers in 2005, to bridge the gap between Computational Mechanics and Computer Aided Design (CAD). The basic IGA paradigm consists of adopting the same basis functions used for geometry representations in CAD systems - such as, e.g., Non-Uniform Rational B-Splines (NURBS) - for the approximation of field variables, in an isoparametric fashion. This leads to a cost-saving simplification of the typically expensive mesh generation and refinement processes required by standard finite element analysis. Thanks to the high-regularity properties of its basis functions, IGA has shown a better accuracy per-degree-of-freedom and an enhanced robustness with respect to standard finite elements in a number of applications ranging from solids and structures to fluids, opening also the door to geometrically flexible discretizations of higher-order partial differential equations in primal form. In particular, the superiority of IGA over standard finite elements appears to be remarkably evident in the approximation of spectra (e.g., in the case of structural vibration studies) and dynamics problems. This lecture aims at giving an overview of the basic features of IGA and of its main advantages. These are illustrated through some convincing applications, mainly belonging to the field of solid and structural mechanics. In particular, after an introduction about the IGA approximation properties of structural vibrations, the application to a real-life case, the so-called NASA “Aluminum Testbed Cylinder”, is shown along with comparisons with experimental results. As a further example, a demanding explicit structural dynamics simulation of a patient-specific aortic valve, modeled by nonlinear hyperelastic shells and involving large deformations and contact, is presented and carefully analyzed in terms of accuracy and efficiency. As a third representative case study, the bending behavior of complex structures like shape memory alloy stents is simulated in the large deformation regime, with particular attention to a correct modeling of buckling phenomena. In all these cases, the superior results which can be provided by isogeometric analysis with respect to standard finite elements are clearly pointed out. Finally, some further work in progress is briefly presented.

**Matteo Rinaldi** :: Carnegie Mellon University

***Slow Motion of the nonlocal Allen-Cahn equation in  $N$  dimensions***

We study the slow motion of solutions of the nonlocal Allen-Cahn equation in a bounded domain  $\Omega \subset \mathbb{R}^N$ , for  $N > 1$ . The initial data is assumed to be close to a configuration whose interface separating the states minimizes the surface area (or perimeter); both global and local perimeter minimizers are taken into account.

We demonstrate that interfaces evolve on a timescale  $\varepsilon^{-1}$ , where  $\varepsilon$  is the interaction length parameter. The key tool is a second order  $\Gamma$ -convergence analysis of the energy functional, which allows us to provide sharp energy estimates. Slow motion of solutions for the Cahn-Hilliard equation starting close to global perimeter minimizers is provided as well.

This is a joint work with Ryan Murray (Carnegie Mellon University).

**Henry Shum** :: University of Pittsburgh

***Chemically powered convective transport and organization of microparticles***

In living cells, fundamental active processes, such as protein synthesis, active transport and cell motility, are achieved and controlled through biochemical reaction networks. A challenge in designing artificial self-propelled or self-organizing microdevices is to make these systems operate autonomously like their biological counterparts using locally available chemical fuel instead of externally controlled power sources. We present two examples of systems that exploit physical and chemical processes to drive long range transport. We employ a combination of theoretical analysis, based on the Greens function for diffusion problems, and numerical simulation. In the first example, we utilize a bioinspired negative feedback cycle with three chemical components known as the repressilator. Chemical oscillations are generated by three microcapsules that each release a different component of the repressilator network. The released species are adsorbed onto the underlying surface and modify its adhesive properties. We show that gradients

in the chemical concentrations can lead to a chemotactic attraction between capsules, causing spontaneous aggregation. Based on results from stability analysis, we predict and verify parameters that allow the microcapsules to turn off once aggregation is complete. This exemplifies self-regulation, which is one of the hallmarks of biological organisms. The second system we explore consists of an immobilized patch of enzyme at the bottom of a fluid filled chamber. This patch catalyzes a chemical reaction and as a result, it acts as a pump that drives fluid convection. Depending on the difference in density between the reaction substrate and product species, the chemical reaction causes buoyancy effects that either push the fluid, and any cargo carried by the fluid, away from the enzyme pump or draw the fluid in. We show that if the substrate and product have different diffusion coefficients, the pump could exhibit much more complex behavior, such as pushing fluid outwards at early times and pulling fluid inwards later on. Two parameters, namely, the ratio of solutal expansion coefficients and the ratio of diffusion coefficients, determine the pump dynamics. In conclusion, the techniques applied to analyze these two systems could aid in the design of chemically responsive functional components within a microfluidic device.

**Francesco Solombrino** :: Technische Universität München

*Frustrated ferromagnetic spin chains: a variational approach to chirality transitions*

We study the energy per particle of a one-dimensional ferromagnetic/anti-ferromagnetic frustrated spin chain with nearest and next-to-nearest interactions close to the helimagnet/ferromagnet transition point as the number of particles diverges.

We rigorously prove the emergence of chiral ground states and we compute, by performing the  $\Gamma$ -limits of proper renormalizations and scalings, the energy for a chirality transition.

This is a joint work with M. Cicalese (TU München).

**Ulisse Stefanelli** :: Universität Wien

*Finite plasticity based on the plastic metric tensor*

A classical approach to finite plastic deformations prescribes the deformation gradient of the medium to be multiplicatively decomposed as  $FP$  where  $F$  stands for the elastic and  $P$  for the plastic strain, respectively. The requirement of frame-indifference imposes that the hyperelastic stored energy density is given in terms of the symmetric Cauchy-Green tensor  $F^t F$  only. Moving from this fact, I shall comment on the possibility of formulating the full finite plasticity model in terms of the corresponding plastic metric tensor  $P^t P$ . This situation is indeed common in applications and bears some relevant advantages with respect to the classical formulation in terms of  $P$ .

The global existence of quasistatic evolutions for this model will be proved. Moreover, I will comment on the possibility of rigorously ascertain the small-deformation, linear limit in terms of evolutive  $\Gamma$ -convergence techniques.

This work is in collaboraton with D. Grandi (Vienna).

**Barbara Zwicknagl** :: Universität Bonn

*Analytical study of morphology of epitaxially strained films: Role of the miscut angle*

I will report analytical results on variational models introduced in the physical literature to describe the shape of an epitaxially deposited film on a rigid substrate in the case in which there are a crystallographic miscut and a mismatch between the lattice parameters of the substrate and the film. The nonlocal isoperimetric energy functionals typically account for the elastic energy stored in the film due to the crystallographic misfit, and an anisotropic surface energy of the film's free surface. I will discuss the influence of a non-vanishing miscut angle, in particular in view of existence of minimizers and their geometric properties. This talk is partially based on joint works with Irene Fonseca and Aldo Pratelli, and with Peter Bella and Michael Goldman.

## Registered participants

1. Elalami Abdessamad - Moulay Ismail University, Meknes
2. Ahmed Aberqi - Sidi Mohammed Ben Abdellah University, Fez
3. Youssef Ahmida - National School of Applied Sciences, Fez
4. Abderrahman Ait Aadi - Moulay Ismail University, Meknes
5. Roberto Alessi - Sapienza Università di Roma
6. Elvis Adam Alhassan - University for Development Studies
7. Stefano Almi - SISSA
8. François Alouges - École Polytechnique, Palaiseau
9. Maurizio Angelillo - Università di Salerno
10. Nenad Antonić - University of Zagreb
11. Veronika Auer - University of Augsburg
12. Francesco Ballarin - SISSA
13. Marco Barchiesi - Università di Napoli Federico II
14. Mohammed Belayachi - Faculté Polydisciplinaire, Taza
15. Rajae Bentahar - ENSA, Fez
16. Dario Bojanjac - University of Zagreb
17. Hamid Bourray - Meknes
18. Farid Bozorgnia - Instituto Superior Técnico, Lisboa
19. Mario Bukal - University of Zagreb
20. Graça Carita - University of Évora
21. Moussa Chrif - C.R.M.E.F, Meknes
22. Vito Crismale - SISSA
23. Riccardo Cristoferi - SISSA
24. Gianni Dal Maso - SISSA
25. Elisa Davoli - Universität Wien
26. Lucia De Luca - Technische Universität München
27. Antonio DeSimone - SISSA
28. Abdeslam El Akkad - CRMEF, Fez
29. Mohamed El Fatini - Ibn Tofail University, Kenitra
30. Youssef El Hadfi - Faculty of science Dhar Mahraz, Fez
31. Radouane El Kinani - Moulay Ismail University, Meknes
32. Ismaili alaoui El Mehdi - Moulay Ismail University, Meknes

33. Abdelkarim Elaida - University Ibn Zohr, Agadir
34. Abella Elkabouss - Moulay Ismail University, Meknes
35. Marko Erceg - University of Zagreb
36. Halima Essaadi - Moulay Ismail University, Meknes
37. Joep Evers - Technische Universiteit Eindhoven
38. Jozsef Farkas - University of Stirling
39. José Augusto Ferreira - Universidade de Coimbra
40. Manuel Friedrich - Universität Wien
41. Hermes Gadêlha - University of York
42. Paolo Gidoni - SISSA
43. Laetitia Giraldi - ENSTA, Paristech
44. Mohamed Hammoumi - Faculty of Science Dhar el Mahraz, Fez
45. Gurgun (Greg) Hayrapetyan - Ohio University
46. Hassane Hjjaj - University Sidi Mohamed Ben Abdellah, Fez
47. Ivan Ivec - University of Zagreb
48. Martin Jesenko - Augsburg University
49. Aleks Jevnikar - SISSA
50. Moustapha Johri - Sultan Moulay Slimane University, Beni Mellal
51. Rasmita Kar - LNMIIT, Jaipur
52. Touria Karite - Moulay Ismail University, Meknes
53. Oouadie Koubaiti - Faculté des Sciences et Techniques, Fez
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55. Rachid Larhrissi - Moulay Ismail University, Meknes
56. Giuliano Lazzaroni - SISSA
57. Ilaria Lucardesi - SISSA
58. Jawad Maekkaoui - Faculté Polydisciplinaire, Beni Mellal
59. Giovanni Maggiani - Università di Pavia
60. Mariam Mahmoudi - National School of Engineers, Monastir
61. Sarangam Majumdar - University of Hamburg
62. Maroje Marohnic - University of Zagreb
63. José Matias - CAMGSD, Instituto Superior Técnico, Universidade de Lisboa
64. Mohamed Mbehou - University of Yaounde 1
65. Mounir Mekkour - FPT Taza, Fez

66. Marin Misur - University of Zagreb
67. Leonard Monsaingeon - CAMGSD, Instituto Superior Técnico, Universidade de Lisboa
68. Maria Giovanna Mora - Università di Pavia
69. Marco Morandotti - SISSA
70. Sara Nabet - Hassan I University, Settat
71. Lorenzo Nardini - SISSA
72. Gianluca Orlando - SISSA
73. Anna Pandolfi - Politecnico di Milano
74. Pablo Pedregal - Universidad de Castilla-La Mancha
75. Paolo Piovano - Universität Wien
76. Alessandro Reali - Università di Pavia
77. Matteo Rinaldi - Carnegie Mellon University
78. Rafael Rojas - Sapienza Università di Roma
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80. Gianluigi Rozza - SISSA
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82. Ahmed Sanhaji - Faculté Polydisciplinaire, Taza
83. Henry Shum - University of Pittsburgh
84. Francesco Solombrino - Technische Universität München
85. Ulisse Stefanelli - Universität Wien
86. Igor Velčić - University of Zagreb
87. Ahmed Youssfi - National School of Applied Sciences, Fez
88. Youssef Zguir - Ibn Tofaill, Kenitra
89. Hayat Zouiten - Moulay Ismail University, Meknes
90. Davide Zucco - SISSA
91. Barbara Zwicknagl - Universität Bonn